**Hypothesis Testing**

**Introduction**

In interval estimation we have discussed the aspect of estimating population parameters based upon sample results.

In this class we will investigate the second aspect of statistical inference.

The purpose of the second aspect of inference is to determine whether a certain assumption or hypothesis about a parameter can be justified by statistical evidence.

Our purpose is to make valid decisions about the population parameters based on the analysis of samples.

In order to make statistical decisions, we make certain assumptions about the population parameters. These assumptions are known as hypothesis. Then a sample is taken to estimate the values of these population parameters. If the estimate favours the hypothesis, then we accept the hypothesis as being correct.

If the value of the sample statistic thus calculated as an estimate of the population parameter does not favour the hypothesis made about the same population parameter, then a decision must be made as to whether this difference is purely a matter of chance which happens in nature (when in-fact the sample statistic and the corresponding population parameter are similar in nature) or whether this difference is significant enough so that it is a real difference and our assumption about the population parameter is not correct.

Since we are testing for our assumptions or *hypothesis* being correct or not, this field of decision making is known as *Hypothesis Testing*.

**Hypothesis Testing**

A claim or hypothesis about the values or population parameters is known as the Null Hypothesis and is written as Ho.

The hypothesis is then tested with the available evidence and the decision is made whether to accept this hypothesis or reject it. If this hypothesis is rejected, then accept the alternative hypothesis. The alternative hypothesis is denoted as H1.

**Accepting / Rejecting**

The process involves testing of the null hypothesis. If the null hypothesis is rejected, then the alternative hypothesis is accepted. Acceptance of the alternative hypothesis does not mean that it is correct. It simply means that there is not enough evidence to be reasonably sure that the null hypothesis is acceptable.

**Errors**

There are two types of errors that can be committed in making decisions regarding accepting or rejecting the null hypothesis. The first type of error is known as Type 1 error. Type 1 error is committed when the null hypothesis is rejected when in fact it is true. The second type of error is known as Type II error. It is committed when a null hypothesis is accepted when in fact it was not true and should have been rejected.

**Hypothesis Testing**

In statistical hypothesis testing and decision making about the values of population parameters as approximated by the sample statistic, the null hypothesis asserts that there is no true difference between the sample statistic and the corresponding population parameter under consideration. If there is any visible difference, it is due to natural fluctuations in sampling.

*The Butler Did It*

Story…

Four Possibilities

1. The Butler is accused by the detective and in fact, he did commit the crime – A correct decision
2. The butler is accused by the detective when in fact, he did not commit the crime – An incorrect decision
3. The butler is considered innocent and in fact, he did not commit the crime – a correct decision
4. The butler is considered innocent and in fact, he did commit the crime – An incorrect decision

|  |  |  |  |
| --- | --- | --- | --- |
| Hypothesis | Fact | Release him | Charge him |
| Ho : Null | Butler Did Not Do It | Correct | Incorrect (I) |
| H1: Alternate | Butler Did It | Incorrect (II) | Correct |

**Summarising**

Ho: An assertion about the population parameter that is being tested by the sample results

H1: A claim about the population parameter that is accepted when the null hypothesis is rejected

Type I Error: An error made in rejecting the null hypothesis when in fact it is true.

Type II Error: An error made in accepting the null hypothesis, when in fact it is false.

**Level of Significance**

Type I error is denoted by α (Alpha) and is expressed as a probability of rejecting a true hypothesis. It is also known as level of significance. (1 – α) expresses the level of confidence.

Type II error is denoted by β and is expressed as the probability of accepting a false hypothesis.

**Procedure for Hypothesis Testing**

1. State the Null hypothesis as well as the alternate hypothesis:

Suppose that we want to test the hypothesis that the average IQ of our college students is 130. Then this would become our null hypothesis and the alternate hypothesis would be that this average IQ is not 130.

μ= 130

μ!= 130

1. Establish a level of significance prior to sampling

The level of significance signifies the probability of committing Type 1 error α and is generally taken as equal to 0.05, which really means that after the hypothesis has been tested and a decision made, we will still be making an error in rejecting the null hypothesis, when in fact it is true 5% of the time. Sometime the value α is established as 0.01.

1. Determine a suitable test statistc.

This means the choice of appropriate probability distribution to use with. Z table or t-table.

1. Define the rejection (critical region)

The critical region will be established on the basis of the choice of the level of significance. For a 5% level of significance, using the standard normal distribution, the value obtained by the sample is not expected to be more than +/- 1.96 s.e.m

If the same sample statistic x̅ falls within 1.96 sem of the assumed value of μunder the assumption of null hypothesis H0, then we accept the null hypothesis as being correct at 95% confidence level.

The difference between x̅ and μwhich may be any value between X1 and μ or X2 and μis considered to be accidental or due to chance element and is not considered significant enough or real enough to reject null hypothesis, so that for all practical purposes the value of x̅ can have any value between X1 and X2 as shown above.

However if the value of x̅ falls beyond X2 on the upper side or beyond X1 on the lower side, then this difference between the values of x̅ and μwould be considered significant and it will lead to rejection of the null hypothesis.

The area of rejection is known as critical region.

1. Making the decision

Accept the null hypothesis if the value of sample statistic x̅ falls within the area of acceptance, otherwise reject the null hypothesis.